

Calc I

Review for derivatives test

answer key

$$\textcircled{1} f(x) = \frac{3x^7 - 6\sqrt{x} + 2\sqrt[5]{x^3}}{2x^2}$$

$$f(x) = \frac{3x^7 - 6x^{\frac{1}{2}} + 2x^{\frac{3}{5}}}{2x^2}$$

$$f(x) = \frac{3}{2}x^5 - 3x^{-\frac{3}{2}} + x^{-\frac{7}{5}}$$

$$f'(x) = \frac{15}{2}x^4 + \frac{9}{2}x^{-\frac{5}{2}} - \frac{7}{5}x^{-\frac{12}{5}}$$

$$f'(x) = \frac{15}{2}x^4 + \frac{9}{2\sqrt{x^5}} - \frac{7}{5\sqrt[5]{x^{12}}}$$

$$\textcircled{2} g(x) = 5x^9 - 7x^2 + 11$$

$$g'(x) = 45x^8 - 14x$$

$$\textcircled{3} f(x) = (2x^3 - 5x)(4x - 3) \Rightarrow f(x) = 8x^4 - 6x^3 - 20x^2 + 15x$$

$$f'(x) = 32x^3 - 18x^2 - 40x + 15$$

$$\textcircled{4} \quad h(x) = \frac{\sin^2(x) + \cos^2(x)}{\csc(x)} \Rightarrow h(x) = \frac{1}{\csc(x)}$$

$$h(x) = \sin(x)$$

$$h'(x) = \cos(x)$$

$$\textcircled{5} \quad f(x) = 3x^5 \cot(x)$$

$$f'(x) = 15x^4 \cot(x) + 3x^5 [-\csc^2(x)]$$

$$f'(x) = 15x^4 \cot(x) - 3x^5 \csc^2(x)$$

$$\textcircled{6} \quad g(x) = (x-3)^4 (3x+2)^2$$

$$g'(x) = 4(x-3)^3 (1) (3x+2)^2 + (x-3)^4 (2(3x+2)(3))$$

$$g'(x) = 4(x-3)^3 (3x+2)^2 + 6(x-3)^4 (3x+2)$$

$$\textcircled{7} \quad h(x) = \frac{3x^3}{\sin(x)}$$

$$h'(x) = \frac{9x^2 \sin(x) - [3x^3 (\cos(x))]}{\sin^2(x)}$$

⑦ (other method)

$$h(x) = \frac{3x^3}{\sin(x)} \Rightarrow h(x) = 3x^3 \csc(x)$$

$$h'(x) = 9x^2 \csc(x) + 3x^3 (-\csc(x) \cot(x))$$

$$h'(x) = 9x^2 \csc(x) - 3x^3 \csc(x) \cot(x)$$

⑧ $f(t) = 3t^2 \tan(4t^3)$

$$f'(t) = 6t \tan(4t^3) + 3t^2 (\sec^2(4t^3) (12t))$$

$$f'(t) = 6t \tan(4t^3) + 36t^3 \sec^2(4t^3)$$

⑨ $g(w) = \sec^4(5w^2) = [\sec(5w^2)]^4$

$$g'(w) = 4[\sec(5w^2)]^3 \cdot [\sec(5w^2) \tan(5w^2)] \cdot [10w]$$

$$g'(w) = 40w \sec^2(5w^2) \tan(5w^2)$$

$$\textcircled{10} \quad h(n) = \sqrt[3]{4x^2-5} \Rightarrow h(n) = (4x^2-5)^{\frac{1}{3}}$$

$$h'(n) = \frac{1}{3} (4x^2-5)^{-\frac{2}{3}} (8x)$$

$$h'(n) = \frac{8x}{3 \sqrt[3]{(4x^2-5)^2}}$$

$$\textcircled{11} \quad g(x) = \sin(\tan(4x))$$

$$g'(x) = [\cos(\tan(4x))] [\sec^2(4x)] [4]$$

$$g'(x) = 4 \sec^2(4x) \cdot \cos(\tan(4x))$$

$$\textcircled{12} \quad h(t) = 4x \cos^3(5x^2) \Rightarrow 4x [\cos(5x^2)]^3$$

$$h'(t) = 4 \cos^3(5x^2) + 4x [3 [\cos(5x^2)]^2 (\sin(5x^2)) \cdot (10x)]$$

$$h'(t) = 4 \cos^3(5x^2) + 120x^2 \cos^2(5x^2) \sin(5x^2)$$

$$\textcircled{13} \quad g(x) = 5x^4 - 7x^3 + 2x - 1$$

$$g'(x) = 20x^3 - 21x^2 + 2 \quad \leftarrow \text{Now take the derivative of this}$$

$$g''(x) = 60x^2 - 42x$$

$$\textcircled{14} \quad f(x) = \frac{3x + 7}{(3x^2 + 2)}$$

$$f'(x) = \frac{3(3x^2 + 2) - 6x(3x + 7)}{(3x^2 + 2)^2}$$

$$f'(x) = \frac{9x^2 + 6 - 18x^2 - 7}{(3x^2 + 2)^2} = \frac{-9x^2 - 1}{(3x^2 + 2)^2} \quad \leftarrow \text{Now take the derivative of this}$$

$$f''(x) = \frac{-18x(3x^2 + 2)^2 - 2(3x^2 + 2)(6x)(-9x^2 - 1)}{[(3x^2 + 2)^2]^2}$$

$$f''(x) = \frac{-18x(3x^2 + 2)^2 - 12x(3x^2 + 2)(-9x^2 - 1)}{(3x^2 + 2)^4}$$

$$\textcircled{15} \quad f(t) = 3t^2 - 5t + 1 \quad \text{at } t = 2$$

$$f'(t) = 6t - 5$$

$$f(2) = 3(2)^2 - 5(2) + 1 = 3 \rightarrow (2, 3)$$

$$f'(2) = 6(2) - 5 = 7 \rightarrow m = 7$$

$$\text{Equation: } (y - 3) = 7(x - 2)$$

$$\textcircled{16} \quad f(x) = 3x \cos^2(2x) \quad \text{at } x = 0$$

$$f'(x) = 3 \cos^2(2x) + 3x [2 \cos(2x)] [-\sin(2x) \cdot (2)]$$

$$f(0) = 3(0) \cos^2(2(0)) = 0 \rightarrow (0, 0)$$

$$f'(0) = 3 \cos^2(2(0)) + 3(0) [2 \cos(2(0))] \cdot [-\sin(2(0)) \cdot (2)]$$

$$3(1) + 3(0) [2[1]] \cdot [-0 \cdot 2]$$

$$3 + 0 = 3 \rightarrow m = 3$$

$$y - 0 = 3(x - 0)$$

$$y = 3x$$

$$(17) \lim_{\Delta x \rightarrow 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$\lim_{\Delta x \rightarrow 0} \frac{-2(x+\Delta x)^2 + 5(x+\Delta x) - 1 - (-2x^2 + 5x - 1)}{\Delta x}$$

$$\lim_{\Delta x \rightarrow 0} \frac{-2(x^2 + 2x(\Delta x) + (\Delta x)^2) + 5x + 5(\Delta x) - 1 + 2x^2 - 5x + 1}{\Delta x}$$

$$\lim_{\Delta x \rightarrow 0} \frac{-\cancel{2x^2} - 4x(\Delta x) - 2(\Delta x)^2 + \cancel{5x} + 5(\Delta x) - \cancel{1} + \cancel{2x^2} - \cancel{5x} + \cancel{1}}{\Delta x}$$

$$\lim_{\Delta x \rightarrow 0} \frac{-4x(\Delta x) - 2(\Delta x)^2 + 5(\Delta x)}{\Delta x}$$

$$\lim_{\Delta x \rightarrow 0} \frac{\cancel{\Delta x}(-4x - 2(\Delta x) + 5)}{\cancel{\Delta x}}$$

$$\lim_{\Delta x \rightarrow 0} (-4x - 2(\Delta x) + 5) = \boxed{-4x + 5}$$